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**STATISTICAL DESCRIPTION OF THE DECONFINING PHASE  
TRANSITION IN A FINITE VOLUME**

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# Abstract

This work deals with a statistical description of a thermally driven deconfining phase transition from a hadronic gas consisting of massless pions to a color-singlet Quark-Gluon Plasma (QGP), in a finite volume. The thermodynamical approach, within the coexistence model is used to investigate the Quantum Chromo-Dynamics deconfining phase transition occurring between the two phases, at vanishing chemical potential. Considering the color singletness condition, the partition function is calculated for the QGP phase with massless up and down quarks, using the group theoretical projection method. The exact total partition function of the studied system is then obtained and employed to calculate mean values of physical quantities, well characterizing the system near the transition. The thermal behavior of the order parameter, the susceptibility, and its derivative with respect to temperature, is investigated, as well as that of the first four cumulants of the probability density, representing respectively: order parameter, variance, skewness and kurtosis. The finite-size effects on the thermally driven deconfining phase transition are studied, and a finite-size scaling analysis shows that the maxima of both susceptibility and variance exhibit a scaling behavior with a power law of the volume characterized by scaling exponents. Based on the fluctuation relationship between the magnetic susceptibility and variance for magnetic finite systems, the correlation between the variance and the thermal susceptibility has been probed for the studied deconfining phase transition. The linearity between the two quantities is obtained in this case also, similarly to the ferromagnetic transition. The correlation between the second-derivative of the order parameter and the third cumulant is also examined.

**Keywords:** Deconfining Phase Transition, QGP, Finite-Size Scaling, Order Parameter, Cumulants, Scaling Exponents.